CSC413/2516 Lecture 11: Q-Learning & the Game of Go

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Overview

- Second lecture on reinforcement learning
  - Optimize a policy directly, don't represent anything about the environment
- Today: Q-learning
  - Learn an action-value function that predicts future returns
- Case study: AlphaGo uses both a policy network and a value network

Finite and Infinite Horizon

- Last time: finite horizon MDPs
  - Fixed number of steps T per episode
  - Maximize expected return $R = \mathbb{E}_{p(\tau)}[r(\tau)]$
- Now: more convenient to assume infinite horizon
  - We can't sum infinitely many rewards, so we need to discount them: \$100 a year from now is worth less than \$100 today
  - Discounted return

$$G_t = r_t + \gamma r_{t+1} + \gamma^2 r_{t+2} + \cdots$$

  - Want to choose an action to maximize expected discounted return
  - The parameter $\gamma < 1$ is called the discount factor
    - small $\gamma =$ myopic
    - large $\gamma =$ farsighted

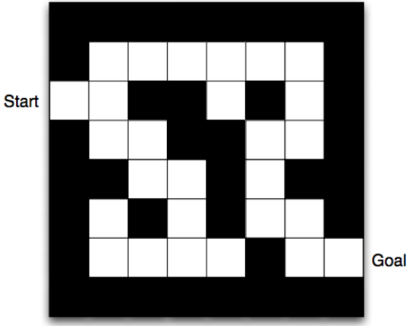
Value Function

- Value function $V^{\pi}(\mathbf{s})$ of a state $\mathbf{s}$ under policy π: the expected discounted return if we start in $\mathbf{s}$ and follow π

$$
\begin{aligned}
V^{\pi}(\mathbf{s}) &= \mathbb{E}[G_t \mid \mathbf{s}_t = \mathbf{s}] \\
&= \mathbb{E}\left[\sum_{i=0}^{\infty} \gamma^i r_{t+i} \mid \mathbf{s}_t = \mathbf{s}\right]
\end{aligned}
$$

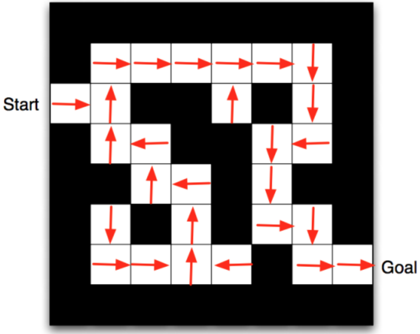
- Computing the value function is generally impractical, but we can try to approximate (learn) it
- The benefit is credit assignment: see directly how an action affects future returns rather than wait for rollouts

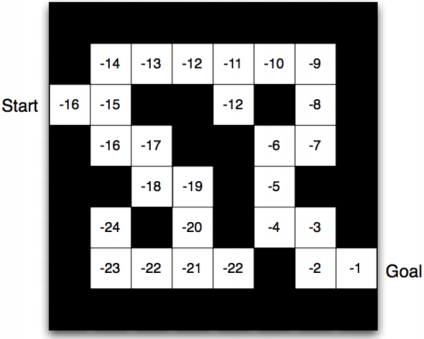
Value Function



- Rewards: -1 per time step
- Undiscounted ($\gamma = 1$)
- Actions: N, E, S, W
- State: current location

Value Function





Action-Value Function

- Can we use a value function to choose actions?

$$\arg\max_{\mathbf{a}} r(\mathbf{s}_t, \mathbf{a}) + \gamma \mathbb{E}_{p(\mathbf{s}_{t+1} \mid \mathbf{s}_t, \mathbf{a}_t)}[V^\pi(\mathbf{s}_{t+1})]$$

Action-Value Function

- Can we use a value function to choose actions?

$$\arg\max_{\mathbf{a}} r(\mathbf{s}_t, \mathbf{a}) + \gamma \mathbb{E}_{p(\mathbf{s}_{t+1} \,|\, \mathbf{s}_t, \mathbf{a}_t)}[V^\pi(\mathbf{s}_{t+1})]$$

- Problem: this requires taking the expectation with respect to the environment's dynamics, which we don't have direct access to!
- Instead learn an action-value function, or Q-function: expected returns if you take action **a** and then follow your policy

$$Q^\pi(\mathbf{s}, \mathbf{a}) = \mathbb{E}[G_t \,|\, \mathbf{s}_t = \mathbf{s}, \mathbf{a}_t = \mathbf{a}]$$

- Relationship:

$$V^\pi(\mathbf{s}) = \sum_{\mathbf{a}} \pi(\mathbf{a} \,|\, \mathbf{s}) Q^\pi(\mathbf{s}, \mathbf{a})$$

- Optimal action:

$$\arg\max_{\mathbf{a}} Q^\pi(\mathbf{s}, \mathbf{a})$$

Bellman Equation

- The Bellman Equation is a recursive formula for the action-value function:

$$Q^\pi(\mathbf{s}, \mathbf{a}) = r(\mathbf{s}, \mathbf{a}) + \gamma \mathbb{E}_{p(\mathbf{s}' \mid \mathbf{s}, \mathbf{a}) \, \pi(\mathbf{a}' \mid \mathbf{s}')}[Q^\pi(\mathbf{s}', \mathbf{a}')]$$

- There are various Bellman equations, and most RL algorithms are based on repeatedly applying one of them.

Optimal Bellman Equation

- The optimal policy π^* is the one that maximizes the expected discounted return, and the optimal action-value function Q^* is the action-value function for π^*.
- The Optimal Bellman Equation gives a recursive formula for Q^*:

$$Q^*(\mathbf{s}, \mathbf{a}) = r(\mathbf{s}, \mathbf{a}) + \gamma \mathbb{E}_{p(\mathbf{s}' \mid \mathbf{s}, \mathbf{a})} \left[\max_{\mathbf{a}'} Q^*(\mathbf{s}_{t+1}, \mathbf{a}') \mid \mathbf{s}_t = \mathbf{s}, \mathbf{a}_t = \mathbf{a} \right]$$

- This system of equations characterizes the optimal action-value function. So maybe we can approximate Q^* by trying to solve the optimal Bellman equation!

Q-Learning

- Let Q be an action-value function which hopefully approximates Q^*.
- The Bellman error is the update to our expected return when we observe the next state $\mathbf{s}'$.

$$\underbrace{r(\mathbf{s}_t, \mathbf{a}_t) + \gamma \max_{\mathbf{a}} Q(\mathbf{s}_{t+1}, \mathbf{a})}_{\text{inside } \mathbb{E} \text{ in RHS of Bellman eqn}} - Q(\mathbf{s}_t, \mathbf{a}_t)$$

- The Bellman equation says the Bellman error is 0 at convergence.
- Q-learning is an algorithm that repeatedly adjusts Q to minimize the Bellman error
- Each time we sample consecutive states and actions $(\mathbf{s}_t, \mathbf{a}_t, \mathbf{s}_{t+1})$:

$$Q(\mathbf{s}_t, \mathbf{a}_t) \leftarrow Q(\mathbf{s}_t, \mathbf{a}_t) + \alpha \underbrace{\left[r(\mathbf{s}_t, \mathbf{a}_t) + \gamma \max_{\mathbf{a}} Q(\mathbf{s}_{t+1}, \mathbf{a}) - Q(\mathbf{s}_t, \mathbf{a}_t) \right]}_{\text{Bellman error}}$$

Exploration-Exploitation Tradeoff

- Notice: Q-learning only learns about the states and actions it visits.
- Exploration-exploitation tradeoff: the agent should sometimes pick suboptimal actions in order to visit new states and actions.
- Simple solution: ϵ-greedy policy
  - With probability $1 - \epsilon$, choose the optimal action according to Q
  - With probability ϵ, choose a random action
- Believe it or not, ϵ-greedy is still used today!

Q-Learning

Initialize $Q(s,a), \forall s \in \mathcal{S}, a \in \mathcal{A}(s)$, arbitrarily, and $Q(\textit{terminal-state}, \cdot) = 0$
Repeat (for each episode):
    Initialize S
    Repeat (for each step of episode):
        Choose A from S using policy derived from Q (e.g., ε-greedy)
        Take action A, observe R, S'
        $Q(S,A) \leftarrow Q(S,A) + \alpha\big[R + \gamma \max_a Q(S',a) - Q(S,A)\big]$
        $S \leftarrow S'$;
    until S is terminal

Function Approximation

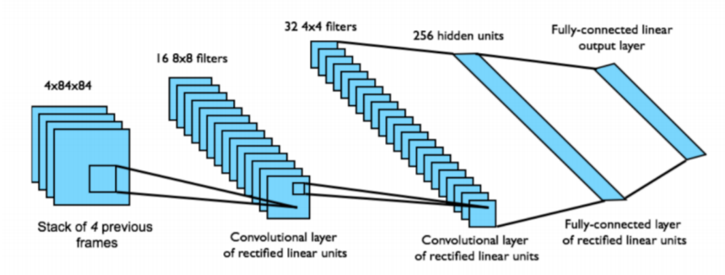
- So far, we've been assuming a tabular representation of Q: one entry for every state/action pair.
- This is impractical to store for all but the simplest problems, and doesn't share structure between related states.
- Solution: approximate Q using a parameterized function, e.g.
  - linear function approximation: $Q(\mathbf{s}, \mathbf{a}) = \mathbf{w}^\top \psi(\mathbf{s}, \mathbf{a})$
  - compute Q with a neural net
- Update Q using backprop:

$$t \leftarrow r(\mathbf{s}_t, \mathbf{a}_t) + \gamma \max_{\mathbf{a}} Q(\mathbf{s}_{t+1}, \mathbf{a})$$

$$\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} + \alpha(t - Q(\mathbf{s}, \mathbf{a})) \frac{\partial Q}{\partial \boldsymbol{\theta}}$$

Function Approximation with Neural Networks

- Approximating Q with a neural net is a decades-old idea, but DeepMind got it to work really well on Atari games in 2013 ("deep Q-learning")
- They used a very small network by today's standards



- Main technical innovation: store experience into a replay buffer, and perform Q-learning using stored experience
  - Gains sample efficiency by separating environment interaction from optimization — don't need new experience for every SGD update!

Policy Gradient vs. Q-Learning

- Policy gradient and Q-learning use two very different choices of representation: policies and value functions
- Advantage of both methods: don't need to model the environment
- Pros/cons of policy gradient
  - Pro: unbiased estimate of gradient of expected return
  - Pro: can handle a large space of actions (since you only need to sample one)
  - Con: high variance updates (implies poor sample efficiency)
  - Con: doesn't do credit assignment
- Pros/cons of Q-learning
  - Pro: lower variance updates, more sample efficient
  - Pro: does credit assignment
  - Con: biased updates since Q function is approximate (drinks its own Kool-Aid)
  - Con: hard to handle many actions (since you need to take the max)

After the break

After the break: **AlphaGo**

Overview

Some milestones in computer game playing:

- 1949 — Claude Shannon proposes the idea of game tree search, explaining how games could be solved algorithmically in principle
- 1951 — Alan Turing writes a chess program that he executes by hand
- 1956 — Arthur Samuel writes a program that plays checkers better than he does
- 1968 — An algorithm defeats human novices at Go

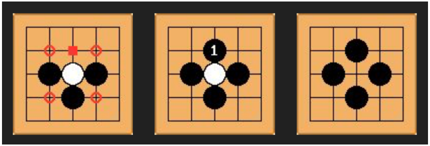
...silence...

- 1992 — TD-Gammon plays backgammon competitively with the best human players
- 1996 — Chinook wins the US National Checkers Championship
- 1997 — DeepBlue defeats world chess champion Garry Kasparov

After chess, Go was humanity's last stand

Go

- Played on a 19×19 board
- Two players, black and white, each place one stone per turn
- Capture opponent's stones by surrounding them



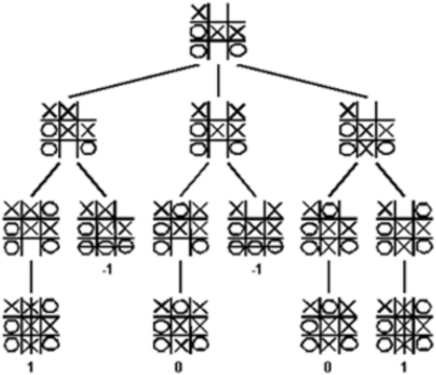
Go

What makes Go so challenging:

- Hundreds of legal moves from any position, many of which are plausible
- Games can last hundreds of moves
- Unlike Chess, endgames are too complicated to solve exactly (endgames had been a major strength of computer players for games like Chess)
- Heavily dependent on pattern recognition

Game Trees

- Each node corresponds to a legal state of the game.
- The children of a node correspond to possible actions taken by a player.
- Leaf nodes are ones where we can compute the value since a win/draw condition was met



https://www.cs.cmu.edu/~adamchik/15-121/lectures/Game%20Trees/Game%20Trees.html

Game Trees

- As Claude Shannon pointed out in 1949, for games with finite numbers of states, you can solve them in principle by drawing out the whole game tree.
- Ways to deal with the exponential blowup
  - Search to some fixed depth, and then estimate the value using an evaluation function
  - Prioritize exploring the most promising actions for each player (according to the evaluation function)
- Having a good evaluation function is key to good performance
  - Traditionally, this was the main application of machine learning to game playing
  - For programs like Deep Blue, the evaluation function would be a learned linear function of carefully hand-designed features

Now for DeepMind's computer Go player, AlphaGo...

Supervised Learning to Predict Expert Moves

- Can a computer play Go without any search?

Supervised Learning to Predict Expert Moves

- Can a computer play Go without any search?
- **Input:** a 19×19 ternary (black/white/empty) image — about half the size of MNIST!
- **Prediction:** a distribution over all (legal) next moves
- **Training data:** KGS Go Server, consisting of 160,000 games and 29 million board/next-move pairs
- **Architecture:** fairly generic conv net
- When playing for real, choose the highest-probability move rather than sampling from the distribution
- This network, which just predicted expert moves, could beat a fairly strong program called GnuGo 97% of the time.
  - This was amazing — basically all strong game players had been based on some sort of search over the game tree

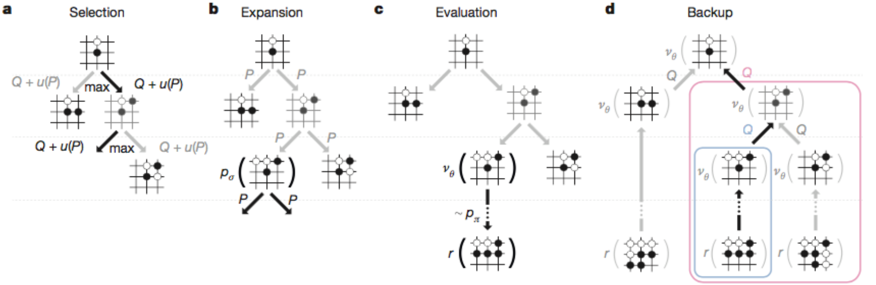
Self-Play and REINFORCE

- The problem from training with expert data: there are only 160,000 games in the database. What if we overfit?
- There is effecitvely infinite data from self-play
  - Have the network repeatedly play against itself as its opponent
  - For stability, it should also play against older versions of itself
- Start with the policy which samples from the predictive distribution over expert moves
  - The network which computes the policy is called the policy network
- REINFORCE algorithm: update the policy to maximize the expected reward r at the end of the game (in this case, $r = +1$ for win, -1 for loss)
- If $\boldsymbol{\theta}$ denotes the parameters of the policy network, a_t is the action at time t, and s_t is the state of the board, and z the rollout of the rest of the game using the current policy

$$R = \mathbb{E}_{a_t \sim p_{\boldsymbol{\theta}}(a_t \mid s_t)}[\mathbb{E}[r(z) \mid s_t, a_t]]$$

Monte Carlo Tree Search

- In 2006, computer Go was revolutionized by a technique called Monte Carlo Tree Search.

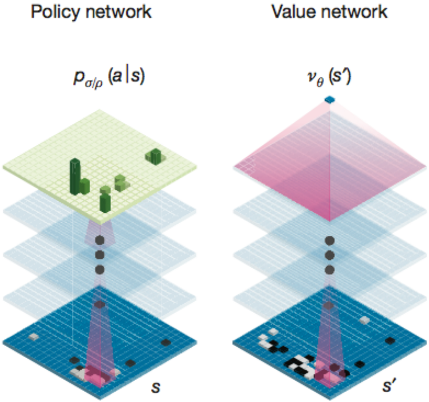


Silver et al., 2016

- Estimate the value of a position by simulating lots of rollouts, i.e. games played randomly using a quick-and-dirty policy
- Keep track of number of wins and losses for each node in the tree
- Key question: how to select which parts of the tree to evaluate?

Tree Search and Value Networks

- We just saw the policy network. But AlphaGo also has another network called a value network.
- This network tries to predict, for a given position, which player has the advantage.
- This is just a vanilla conv net trained with least-squares regression.
- Data comes from the board positions and outcomes encountered during self-play.



Silver et al., 2016

Policy and Value Networks

- AlphaGo combined the policy and value networks with Monte Carlo Tree Search
- Policy network used to simulate rollouts
- Value network used to evaluate leaf positions

AlphaGo Timeline

- **Summer 2014** — start of the project (internship project for UofT grad student Chris Maddison)
- **October 2015** — AlphaGo defeats European champion
  - First time a computer Go player defeated a human professional without handicap — previously believed to be a decade away
- **January 2016** — publication of Nature article "Mastering the game of Go with deep neural networks and tree search"
- **March 2016** — AlphaGo defeats gradmaster Lee Sedol
- **October 2017** — AlphaGo Zero far surpasses the original AlphaGo without training on any human data
- **Decemter 2017** — it beats the best chess programs too, for good measure

AlphaGo

Further reading:

- Silver et al., 2016. Mastering the game of Go with deep neural networks and tree search. *Nature* http://www.nature.com/nature/journal/v529/n7587/full/nature16961.html
- Scientific American: https://www.scientificamerican.com/article/how-the-computer-beat-the-go-master/
- Talk by the DeepMind CEO: https://www.youtube.com/watch?v=aiwQsa_7ZIQ&list=PLqYmG7hTraZCGIymT8wVVIXLWkKPNBoFN&index=8